APPLIED MATHEMATICS-II

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Unit-5

- Integration
  - Indefinite Integral
  - Definite Integral

Indefinite Integral

Antiderivative

- Inverse of differentiation is called Integration. Suppose F'(x)=f(x), then F(x) is called the integral or antiderivative of f(x).
- In general if F(x) is the antiderivative of f(x) and C is any constant then F(x)+C is also antiderivative of f(x) as

$$\frac{d}{dx}(F(x)+C)=F'(x)=f(x)$$

It means that if f(x) possesses an antiderivative then it possesses infinitely many antiderivatives.

Indefinite Integral

- Let f(x) is any function, then collection of all antiderivatives of f(x) is know as indefinite integral of f(x) and it is denoted by $\int f(x)\,dx$.

Hence $\frac{d}{dx}\left(F(x)+C\right)=f(x)$ iff $\int f(x)\,dx=F(x)+C$

where F(x) is antiderivative of f(x) and C is constant of integration.

Points to remember

$\int \rightarrow$ Sign of integration

$\int f(x)\,dx \rightarrow$ Integration of f(x) with respect to x

Some Basic Formulas:

$$\int x^n\,dx = \frac{x^{n+1}}{n+1} + C,\ where\, n \neq -1$$

$$\int e^x dx = e^x + C$$

$$\int a^x dx = \frac{a^x}{\log a} + C$$

$$\int \frac{1}{x} dx = \log|x| + C$$

$$\int \cos(x)\,dx = \sin(x) + C$$

$$\int \sin(x)\,dx = -\cos(x) + C$$

$$\int \sec^2 x\,dx = \tan(x) + C$$

$$\int \sec(x)\tan(x)\,dx = \sec(x) + C$$

$$\int \operatorname{cosec}^2 x\,dx = -\cot(x) + C$$

$$\int \operatorname{cosec}(x)\cot(x)\,dx = -\operatorname{cosec}(x) + C$$

$$\int \cot(x)\,dx = \log\left|\sin(x)\right| + C$$

$$\int \tan(x)\,dx = -\log\left|\cos(x)\right| + C$$

$$\int \sec(x)\,dx = \log\left|\sec(x) + \tan(x)\right| + C$$

$$\int \operatorname{cosec}(x)\,dx = \log\left|\operatorname{cosec}(x) - \cot(x)\right| + C$$

Some Properties of Integration

$$\frac{d}{dx}\left(\int f(x)\,dx\right) = f(x)$$

$$\int \left(f(x) + g(x)\right) dx = \int f(x)\,dx + \int g(x)\,dx$$

$$\int \left(f(x) - g(x)\right) dx = \int f(x)\,dx - \int g(x)\,dx$$

$$\int K\, f(x)\,dx = K \int f(x)\,dx \quad \text{for some constant K}$$

Some Solved Problems

Q.1. Evaluate $\int \left(x^2 - 2x + 3\right) dx$

Sol. $\int \left(x^2 - 2x + 3\right) dx = \int x^2\, dx - 2\int x\, dx + 3\int dx$

$$= \frac{x^3}{3} - 2\left(\frac{x^2}{2}\right) + 3x + C$$

, where C is constant of integration

$$= \frac{x^3}{3} - x^2 + 3x + C$$

Q.2. Evaluate $\int\left(2^{x}-\frac{3}{x}+\tan(x)-\cos(x)\right)dx$

Sol. $\int\left(2^{x}-\frac{3}{x}+\tan(x)-\cos(x)\right)dx$

$$=\int 2^{x}\,dx-3\int\frac{1}{x}\,dx-\int\cos(x)\,dx$$

$$=\frac{2^{x}}{\log(2)}-3\log|x|-\log|\cos(x)|-\sin(x)+C$$

where C is constant of integration

Integration by Substitution

So many problems of integration can be solved easily by method of substitution. Two fundamental deductions are given below:

1. $\int \frac{f'(x)}{f(x)} dx = \log\left| f(x) \right|$

2. $\int \left(f(x) \right)^n f'(x)\, dx = \frac{\left(f(x) \right)^{n+1}}{n+1} + C \ \ where\, n \neq -1$

Note: Here we substitute f(x)=t

f '(x)dx=dt

and then integrate.

Solution by Substitution Method

Q.1. Evaluate $\int \frac{2x+3}{x^2+3x-8}\,dx$

Sol. $Put\ x^2+3x-8=t$

$$\Rightarrow \left(2x+3\right)dx=dt$$

$$\int \frac{2x+3}{x^2+3x-8}\,dx = \int \frac{dt}{t}$$

$$=\log\left|t\right|+C$$

$$=\log\left|x^2+3x-8\right|+C$$

Q.2. Evaluate $\int \frac{e^x + e^{-x}}{e^x - e^{-x}} dx$

Sol. $Put\ e^x - e^{-x} = t$

$$\Rightarrow \left(e^x + e^{-x}\right) dx = dt$$

$$\int \frac{e^x + e^{-x}}{e^x - e^{-x}} dx = \int \frac{dt}{t}$$

$$= \log|t| + C$$

$$= \log\left| e^x - e^{-x} \right| + C$$

To solve integrals of the form:

$$\int \sin px \cos qx\, dx, \int \cos px \sin qx\, dx,$$

$$\int \cos px \cos qx\, dx \ \& \int \sin px \sin qx\, dx$$

Here we will use the following trigonometric relations:

(i) $2\sin A\cos B = \sin(A+B) + \sin(A-B)$

(ii) $2\cos A\sin B = \sin(A+B) - \sin(A-B)$

(iii) $2\cos A\cos B = \cos(A-B) + \cos(A+B)$

(iv) $2\sin A\sin B = \cos(A-B) - \cos(A+B)$

Example:

Q.1. Evaluate $\int 2\cos 6x \cos 4x \, dx$

Sol. $\int 2\cos 6x \cos 4x \, dx = \int [\cos(6x-4x)+\cos(6x+4x)]dx$

$$= \int [\cos(2x)+\cos(10x)]dx$$

$$= \frac{\sin 2x}{2} + \frac{\sin 10x}{10} + C$$

$$\left[used \int \cos(ax+b)dx = \frac{\sin(ax+b)}{a} + C \right]$$

To solve integrals of the form:

$\int \sin^m x \cos^n x \, dx$, where m and n are positive integers.

1. If m is odd then substitute $\cos x = t$.

2. If n is odd then substitute $\sin x = t$.

3. If both m and n are odd then we can substitute either $\cos x = t$ or $\sin x = t$.

Example:

Q.1. Evaluate $\int \sin^6 x \cos^3 x \, dx$

Sol. $Let\ I = \int \sin^6 x \cos^3 x \, dx$

$$= \int \sin^6 x \cos^2 x \cos x \, dx = \int \sin^6 x \left(1 - \sin^2 x\right) \cos x \, dx$$

$$Put\ \sin x = t, then\ \cos x \, dx = dt$$

$$\therefore \quad I = \int t^6 \left(1 - t^2\right) dt = \int \left(t^6 - t^8\right) dt = \frac{t^7}{7} - \frac{t^9}{9} + C$$

$$= \frac{\sin^7 x}{7} - \frac{\sin^9 x}{9} + C$$

Some Special Integrals

$$\int \frac{1}{\sqrt{a^2-x^2}}dx = \sin^{-1}\left(\frac{x}{a}\right)+C$$

$$\int \frac{-1}{\sqrt{a^2-x^2}}dx = \cos^{-1}\left(\frac{x}{a}\right)+C$$

$$\int \frac{1}{x\sqrt{x^2-a^2}}dx = \frac{1}{a}\sec^{-1}\left(\frac{x}{a}\right)+C$$

$$\int \frac{-1}{x\sqrt{x^2-a^2}}dx = \frac{1}{a}\cos ec^{-1}\left(\frac{x}{a}\right)+C$$

$$\int \frac{1}{x^2+a^2}dx = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)+C$$

$$\int \frac{-1}{x^2+a^2}dx = \frac{1}{a}\cot^{-1}\left(\frac{x}{a}\right)+C$$

$$\int \frac{1}{\sqrt{x^2+a^2}}\,dx = \log\left|x+\sqrt{x^2+a^2}\right| + C$$

$$\int \frac{1}{\sqrt{x^2-a^2}}\,dx = \log\left|x+\sqrt{x^2-a^2}\right| + C$$

$$\int \frac{1}{x^2-a^2}\,dx = \frac{1}{2a}\log\left|\frac{x-a}{x+a}\right| + C$$

$$\int \frac{1}{a^2-x^2}\,dx = \frac{1}{2a}\log\left|\frac{a+x}{a-x}\right| + C$$

Examples:

Q.1. Evaluate $\displaystyle\int \frac{1}{9+16x^2}\,dx$

Sol. $$\int \frac{1}{9+16x^2}\,dx = \frac{1}{16}\int \frac{1}{x^2+9/16}\,dx = \frac{1}{16}\int \frac{1}{x^2+(3/4)^2}\,dx$$

$$= \frac{1}{16}\times\frac{1}{3/4}\tan^{-1}\left(\frac{x}{3/4}\right)+C$$

$$= \frac{1}{12}\tan^{-1}\left(\frac{4x}{3}\right)+C$$

Q.2. Evaluate $\int \frac{1}{\sqrt{9x^2-4}}\,dx$

Sol. $\int \frac{1}{\sqrt{9x^2-4}}\,dx = \frac{1}{3}\int \frac{1}{\sqrt{x^2-4/9}}\,dx = \frac{1}{3}\int \frac{1}{\sqrt{x^2-(2/3)^2}}\,dx$

$$= \frac{1}{3}\log\left|x+\sqrt{x^2-(2/3)^2}\right|+C$$

$$= \frac{1}{3}\log\left|x+\sqrt{x^2-4/9}\right|+C$$

Product Rule of Integration

$$\int u\,v\,dx = u\int v\,dx - \int\left(\frac{du}{dx}\int v\,dx\right)dx$$

here u and v both are functions of 'x'. u is the first function and v is the second function.

Tips for choosing 1st and 2nd function

We choose first function which appears first in the word

I L A T E

Inverse Trigonometry Functions
e.g. $\sin^{-1}x$,
$\cos^{-1}x$

Logarithmic Functions
e.g. log(x)

Algebraic Functions
e.g. x, x^3+2

Trigonometry Functions
e.g. cos(2x),
tan(x)

Exponential Functions
e.g. exp(x)

Some Problems on Product Rule

Q.1. Evaluate $\int x \cos 2x \, dx$

Sol. $\int \underset{I}{x} \underset{II}{\cos} 2x \, dx = x \int \cos 2x \, dx - \int \left(\frac{dx}{dx} \int \cos 2x \, dx \right) dx$

$$= x \frac{\sin 2x}{2} - \int \frac{\sin 2x}{2} dx + C$$

$$= \frac{x \sin 2x}{2} + \frac{\cos 2x}{4} + C$$

Q.2. Evaluate $\int x e^{-2x}\,dx$

Sol. $$\int \underset{I}{x}\,\underset{II}{e^{-2x}}\,dx = x\int e^{-2x}\,dx - \int\left(\frac{dx}{dx}\int e^{-2x}\,dx\right)dx$$

$$= x\frac{e^{-2x}}{-2} - \int\frac{e^{-2x}}{-2}\,dx + C$$

$$= \frac{-x e^{-2x}}{2} - \frac{e^{-2x}}{4} + C$$

To solve integrals of the form:

$$\int e^x \left(f(x) + f'(x) \right) dx$$

$$\begin{aligned}
\int e^x \left(f(x) + f'(x) \right) dx &= \int e^x f(x)\, dx + \int e^x f'(x)\, dx \\
&= f(x) \int e^x\, dx - \int \left(f'(x) \int e^x\, dx \right) dx + \int e^x f'(x)\, dx \\
&= f(x) e^x - \int f'(x) e^x\, dx + \int e^x f'(x)\, dx + C \\
&= f(x) e^x + C
\end{aligned}$$

Example

Q.1. Evaluate $\int e^x\left(\sin^{-1}x+\frac{1}{\sqrt{1-x^2}}\right)dx$

Sol. $\int e^x\left(\sin^{-1}x+\frac{1}{\sqrt{1-x^2}}\right)dx$

$$=\sin^{-1}x\int e^x\,dx-\int\left(\frac{d\sin^{-1}x}{dx}\int e^x\,dx\right)dx+\int e^x\frac{1}{\sqrt{1-x^2}}dx$$

$$=\sin^{-1}x\,e^x-\int\frac{1}{\sqrt{1-x^2}}e^x dx+\int e^x\frac{1}{\sqrt{1-x^2}}dx+C$$

$$=\sin^{-1}x\;e^x+C$$

Definite Integral

Definite Integral

- Let f(x) be any function and F(x) be the antiderivative of f(x) defined on [a , b], then

$$\int_a^b f(x)\,dx = \left[\,F(x)\,\right]_a^b = F(a) - F(b)$$

is called the definite integral of f(x) with respect to x from x=a to x=b.

- ‘a’ is called the lower limit of integration and ‘b’ is called the upper limit of integration.
- Note: While evaluating definite integral, arbitrary constant ‘C’ is not added as it cancels automatically.

Some Solved Problems

Q.1. Evaluate $\int_1^3 \left(-x^2 + 2x + \sqrt{x}\right) dx$

Sol. $\int_1^3 \left(-x^2 + 2x + \sqrt{x}\right) dx = \left[-\frac{x^3}{3} + x^2 + \frac{2}{3} x^{\frac{3}{2}}\right]_1^3$

$$= \left[-\frac{3^3}{3} + 3^2 + \frac{2}{3} 3^{\frac{3}{2}}\right] - \left[-\frac{1^3}{3} + 1^2 + \frac{2}{3} 1^{\frac{3}{2}}\right]$$

$$= -2\sqrt{3} - \frac{4}{3}$$

Q.2. Evaluate $\int_{-3}^{-1} \frac{1}{x}\,dx$

Sol. $\int_{-3}^{-1} \frac{1}{x}\,dx = \left[\log|x|\right]_{-3}^{-1} = \log|-1| - \log|-3| = \log 1 - \log 3 = \log\left(\frac{1}{3}\right)$

Q.3. Evaluate $\int_{0}^{1} \frac{1}{\sqrt{1-x^2}}\,dx$

Sol. $\int_{0}^{1} \frac{1}{\sqrt{1-x^2}}\,dx = \left[\sin^{-1} x\right]_0^1$

$$= \sin^{-1} 1 - \sin^{-1} 0$$

$$= \frac{\pi}{2} - 0 = \frac{\pi}{2}$$

Some Standard Results

1. To evaluate $\int_0^{\pi/2} \sin^n x\,dx$ *or* $\int_0^{\pi/2} \cos^n x\,dx$

   where n is positive integer.

Case (i) : When n is an odd positive integer

$$\int_0^{\pi/2} \sin^n x\,dx = \int_0^{\pi/2} \cos^n x\,dx = \frac{(n-1)(n-3)\ldots upto\ positive\ factor}{n(n-2)\ldots upto\ positive\ factor}$$

Case (ii) : When n is an even positive integer

$$\int_0^{\pi/2} \sin^n x\,dx = \int_0^{\pi/2} \cos^n x\,dx = \frac{\pi}{2}\,\frac{(n-1)(n-3)\ldots upto\ positive\ factor}{n(n-2)\ldots upto\ positive\ factor}$$

2. To evaluate $\int_0^{\pi/2} \sin^n x \cos^m x \, dx$

where n and m both are positive integer.

Case (i) : When either n or m or both are odd.

$$\int_0^{\pi/2} \sin^n x \cos^m x \, dx$$

$$= \frac{((n-1)(n-3)\ldots upto +ve\ factor)((m-1)(m-3)\ldots upto +ve\ factor)}{(n+m)(n+m-2)\ldots upto\ positive\ factor}$$

Case (ii) : When either n or m or both are odd.

$$\int_0^{\pi/2} \sin^n x \cos^m x \, dx$$

$$= \frac{\pi}{2} \frac{((n-1)(n-3)\ldots upto +ve\ factor)((m-1)(m-3)\ldots upto +ve\ factor)}{(n+m)(n+m-2)\ldots upto\ positive\ factor}$$

Some Solved Problems

Q.1. Evaluate $\int_0^{\pi/2} \cos^5 x\,dx$

Sol. $\int_0^{\pi/2} \cos^5 x\,dx = \frac{4\times 2}{5\times 3\times 1} = \frac{8}{15}$

Q.2. Evaluate $\int_0^{\pi/2} \sin^6 x\,dx$

Sol. $\int_0^{\pi/2} \sin^6 x\,dx = \frac{5\times 3\times 1}{6\times 4\times 2} = \frac{15}{48} = \frac{5}{16}$

Q.3. Evaluate $\int_0^{\pi/2} \sin^6 x \cos^5 x \, dx$

Sol. $\int_0^{\pi/2} \sin^6 x \cos^5 x \, dx = \dfrac{5\times3\times1\times4\times2}{11\times9\times7\times5\times3\times1} = \dfrac{8}{693}$

Q.4. Evaluate $\int_0^{\pi/2} \sin^4 x \cos^6 x \, dx$

Sol. $\int_0^{\pi/2} \sin^4 x \cos^6 x \, dx = \dfrac{\pi}{2} \dfrac{3\times1\times5\times3\times1}{10\times8\times6\times4\times2} = \dfrac{3\pi}{512}$

Q.5. Evaluate $\int_0^{\pi/2} \sin^5 x \cos^3 x \, dx$

Sol. $\int_0^{\pi/2} \sin^5 x \cos^3 x \, dx = \dfrac{4\times2\times2}{8\times6\times4\times2} = \dfrac{1}{24}$

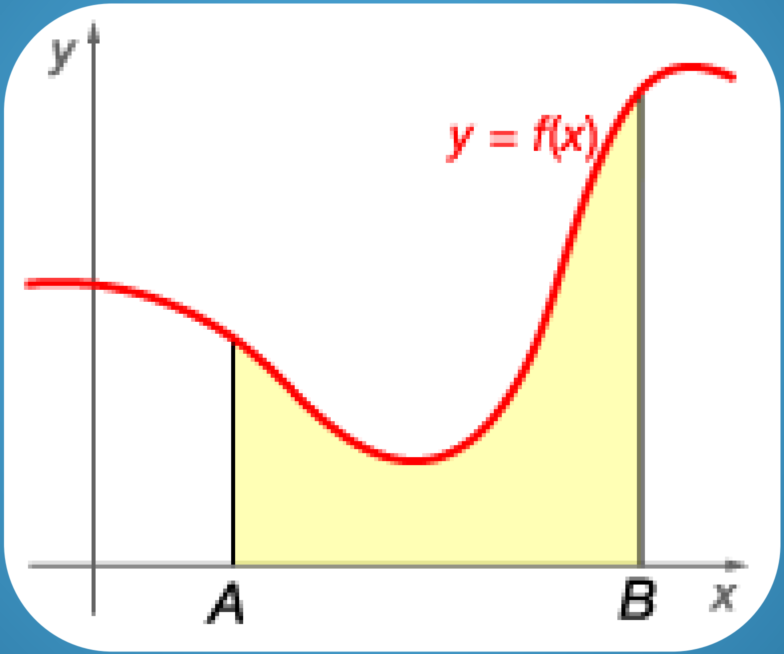
Applications of Integration

Definite Integral as Area Under a Curve

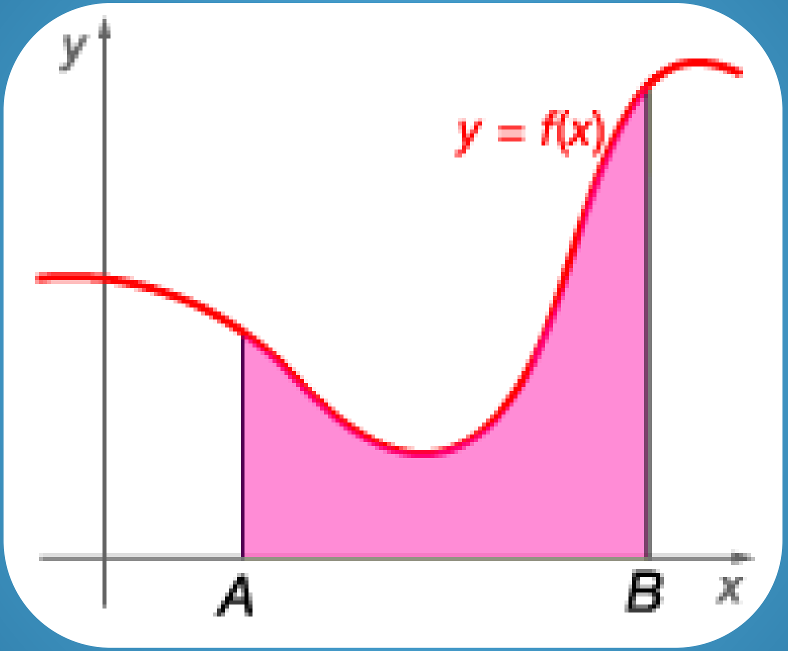
- If f(x) be a continuous non-negative function in [a , b]. Then the area bounded by the curve y=f(x), the ordinates x=a, x=b and the x-axis is given by

$$\int_a^b f(x)\,dx$$

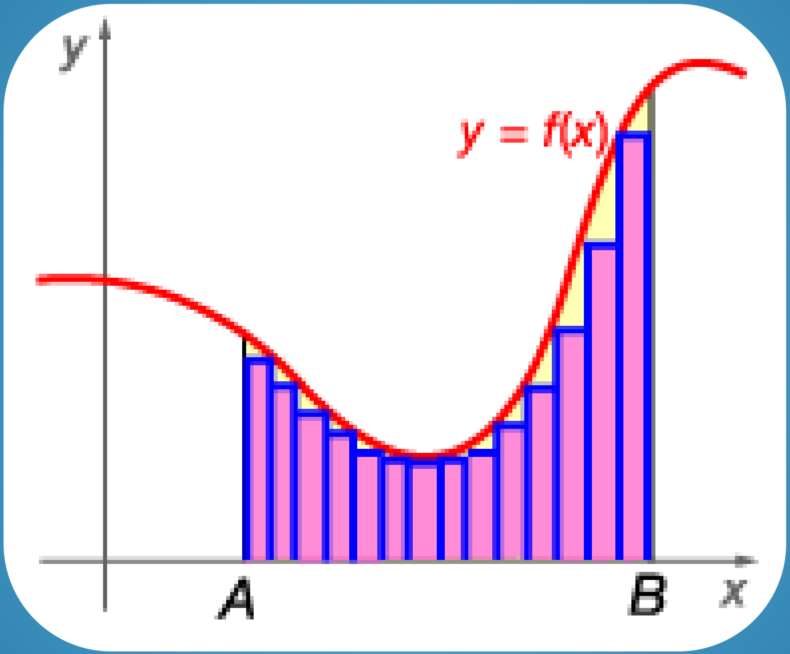
Area Under the Curve



How to find the area under a curve,
between x=A, x=B and
above the x-axis?

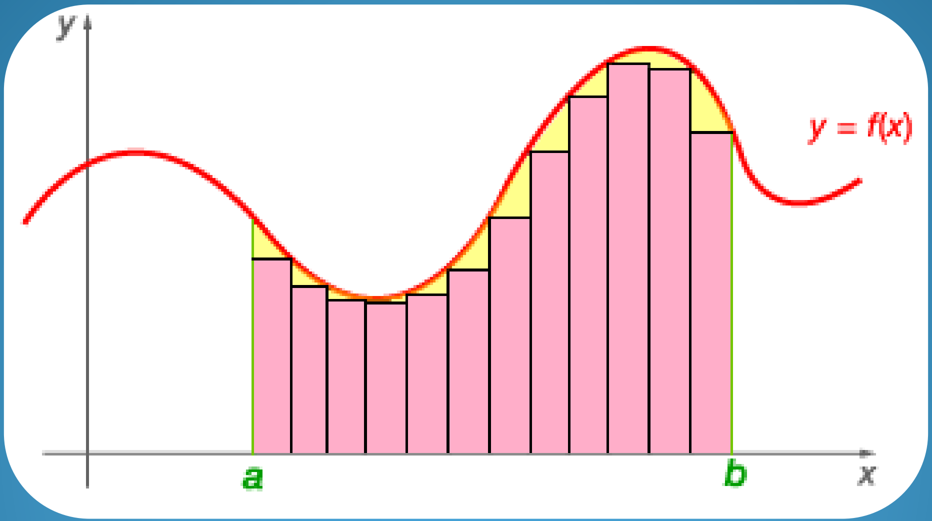


It is possible to find the exact area by letting the width of each rectangle approach zero. Doing this generates an infinite number of rectangles.



$$\text{Area} = R_1 + R_2 + \ldots + R_{13} + R_{14} = \sum_{i=1}^{14} R_i$$

As the number of rectangles used to approximate the area of the region increases, the approximation becomes more accurate.



Area ≈ sum of the areas of the rectangles

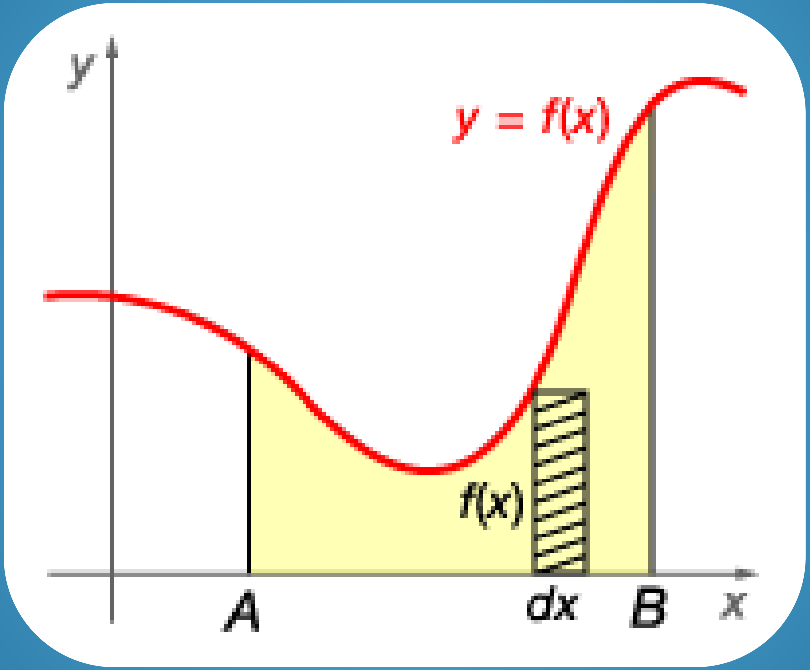
$=\int$um (height) $\bullet$ (base)

$$= \int f(x)\,\Delta x$$

$$= \int_a^b f(x)\,dx$$

The formula looks like an integral.

The Fundamental Theorem of Calculus



$$\text{Area} = \int_A^B f(x)\,dx = F(B) - F(A)$$

$$\text{where } F'(x) = f(x)$$

Conclusion

- As the number of rectangles used to approximate the area of the region increases, the approximation becomes more accurate.

- It is possible to find the exact area by letting the width of each rectangle approach zero. Doing this generates an infinite number of rectangles.

- The Fundamental Theorem of Calculus enables us to evaluate definite integrals. This empowers us to find the area between a curve and the x-axis.

Some Solved Problems

Q.1. Evaluate the area under the curve y= 2x+1, between the ordinates x=0, x=2 and the x-axis.

Sol. The given curve is y= 2x+1. Also y is continues non-negative function between x=0 and x=2.

Thus the curve lies above the x-axis.

So, Required Area= $\int_0^2 y\,dx = \int_0^2 (2x+1)\,dx = \left[2\frac{x^2}{2} + x\right]_0^2$

$$= \left[2\left(\frac{2^2}{2}\right) + 2\right] - 0 = 6\,sq.units$$

Q.2. Find the area under the curve $y^2=x$, between the ordinates $x=1$, $x=4$ and the x-axis in the first quadrant.

Sol. $y^2=x$ is a right handed parabola which is symmetric about x-axis. Area bounded by this parabola between $x=1$ and $x=4$ in first quadrant is given by:

$$\int_1^4 y\,dx=\int_1^4 \sqrt{x}\,dx=\left[\frac{x^{3/2}}{3/2}\right]_1^4$$

$$=\left[\frac{2}{3}x^{3/2}\right]_1^4=\left[\frac{2}{3}\left(4^{3/2}\right)\right]-\left[\frac{2}{3}\left(1^{3/2}\right)\right]$$

$$=\frac{16}{3}-\frac{2}{3}=\frac{14}{3}\,sq.units$$

Numerical Integration

- Numerical integration is a process to evaluate the definite integral from the tabulated vales of the Integrand. We will the following rules to find the approximate area under the curve:

1. Trapezoidal Rule
2. Simpson's Rule

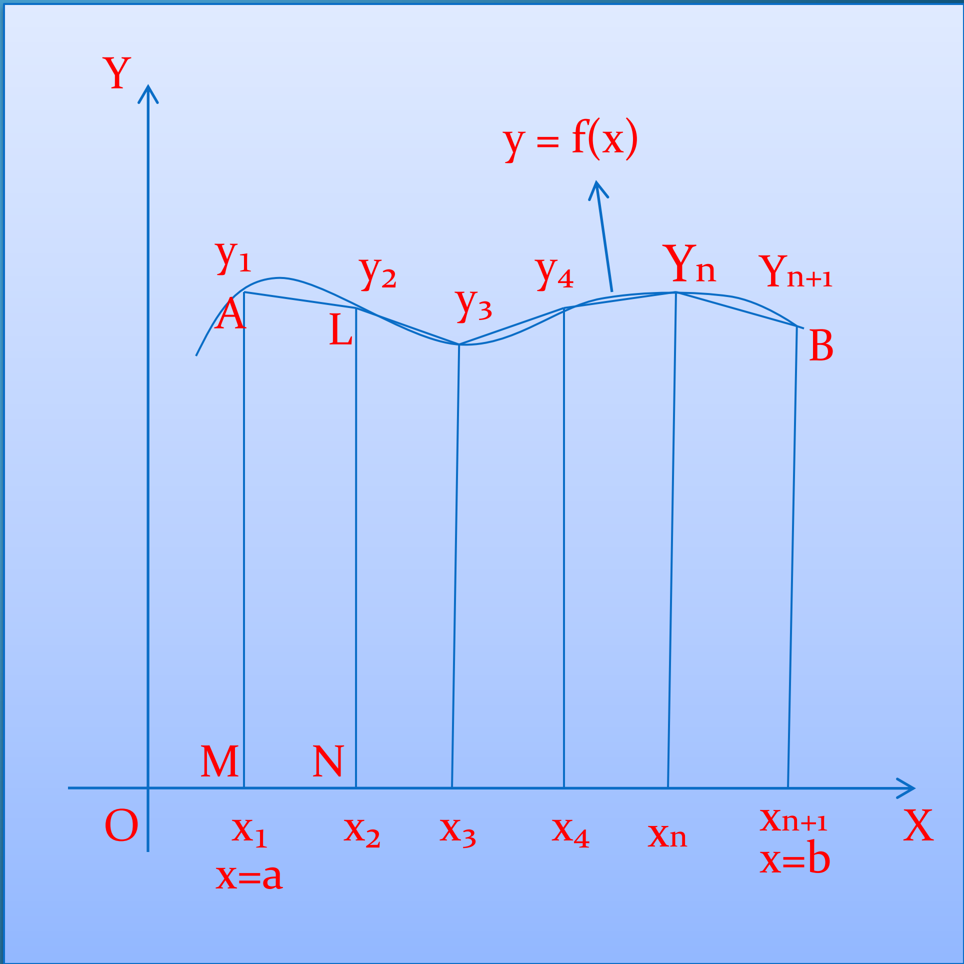
1. Trapezoidal Rule

Let AB be the graph of the curve y=f(x) between x=a and x=b.

Divide the interval [a , b] into n equal subintervals.

So width of each subinterval is $h = \dfrac{b-a}{n}$.

We get n+1 values of x on x-axis i.e. $x_1, x_2, x_3, \ldots, x_n, x_{n+1}$ and there corresponding n+1 values of y i.e. $y_1, y_2, y_3, \ldots, y_n, y_{n+1}$



Now area under AP is approximately equal to the area of the Trapezium ALNM $= \frac{h}{2}(y_1 + y_2)$

So, Total area under the curve AB is given by

$$\int_a^b y\,dx = \text{Sum of Areas of all Trapeziums}$$

$$= \frac{h}{2}(y_1 + y_2) + \frac{h}{2}(y_2 + y_3) + \ldots + \frac{h}{2}(y_n + y_{n+1})$$

$$= \frac{h}{2}\left[(y_1 + y_{n+1}) + 2(y_2 + y_3 + \ldots + y_n)\right]$$

$$\therefore \quad \text{Re}\,quired\;\;Area = \int_a^b y\,dx = \frac{h}{2}\left[(y_1 + y_{n+1}) + 2(y_2 + y_3 + \ldots + y_n)\right]$$

2. Simpson's Rule

Let AB be the graph of the curve y=f(x) between x=a and x=b.

Divide the interval [a , b] into 2n equal subintervals.

So width of each subinterval is $h = \frac{b-a}{n}$.

As number of intervals is even so that the number of ordinates is odd.

$y_1, y_2, y_3, \ldots, y_{2n}, y_{2n+1}$ be the ordinates.

The approximate area under the curve is given by:

$$\int_a^b y\,dx = \frac{h}{3}\left[\left(y_1 + y_{2n+1}\right) + 2\left(y_3 + y_5 + \ldots + y_{2n-1}\right) + 4\left(y_2 + y_4 + \ldots + y_{2n}\right)\right]$$

Observations:

- In the Simpson’s one third rule n is always even.
- In the Trapezoidal rule n may be even or odd.
- The results given by Simpson’s rule are more accurate than the results given by Trapezoidal rule.
- n = number of subintervals
- a = lower limit
- b = upper limit
- $h = \dfrac{b-a}{n}$

Some Solved Problems

Q.1. Find the area under the curve $y = x^2$ by Trapezoidal Rule using 7 equal intervals where $0 \le x \le 7$.

Ans. Here $y = x^2, \quad a = 0, \quad b = 7, \quad n = 7$

$$h = \frac{b-a}{n} = \frac{7-0}{7} = 1$$

$$x_1 = a = 0$$
$$x_2 = a + h = 0 + 1 = 1$$
$$x_3 = a + 2h = 0 + 2 = 2$$
$$x_4 = a + 3h = 0 + 3 = 3$$
$$x_5 = a + 4h = 0 + 4 = 4$$
$$x_6 = a + 5h = 0 + 5 = 5$$
$$x_7 = a + 5h = 0 + 6 = 6$$
$$x_8 = a + 6h = 0 + 7 = 7$$

$$y_1 = x_1^2 = 0^2 = 0 \qquad y_5 = x_5^2 = 4^2 = 16$$

$$y_2 = x_2^2 = 1^2 = 1 \qquad y_6 = x_6^2 = 5^2 = 25$$

$$y_3 = x_3^2 = 2^2 = 4 \qquad y_7 = x_7^2 = 6^2 = 36$$

$$y_4 = x_4^2 = 3^2 = 9 \qquad y_8 = x_8^2 = 7^2 = 49$$

x	0	1	2	3	4	5	6	7
y	0	1	4	9	16	25	36	49

<u>Using Trapezoidal rule:</u>

Required Area $= \int_0^7 x^2\,dx = \frac{h}{2}\left[(y_1 + y_8) + 2(y_2 + y_3 + y_4 + y_5 + y_6 + y_7)\right]$

$$= \frac{1}{2}\left[(0+49) + 2(1+4+9+16+25+36)\right] = \frac{231}{2} = 115.5$$

Q.2. Evaluate $\int_0^1 \frac{1}{1+x}dx$ by Trapezoidal rule using 8 equal Intervals.

Ans. Here $y = \frac{1}{1+x}, \quad a = 0, \quad b = 1, \quad n = 8$

$$h = \frac{b-a}{n} = \frac{1-0}{8} = 0.125$$

x	0	0.125	0.250	0.375	0.500	0.625	0.750	0.875	1
y	1	0.8889	0.8	0.7273	0.6667	0.6154	0.5714	0.5333	0.5

Using Trapezoidal rule:

Required Area $=\int_0^1 \frac{1}{1+x}\,dx$

$$=\frac{0.125}{2}\left[\left(1+0.5\right)+2\left(0.8889+0.8+0.7273+0.6667+0.6154+0.5714+0.5333\right)\right]$$

$$=\frac{0.125}{2}\left[1.500+2\left(4.803\right)\right]$$

$$=\frac{0.125}{2}\left[11.106\right]$$

$$=\frac{1.38825}{2}=0.6941$$

Q.3. Evaluate $\int_{1}^{9}\left(x^2+1\right)dx$ by Simpson's rule using 8 equal Intervals.

Ans. Here $y=x^2+1, \quad a=1, \quad b=9, \quad n=8$

$$h=\frac{b-a}{n}=\frac{9-1}{8}=1$$

x	1	2	3	4	5	6	7	8	9
y	2	5	10	17	26	37	50	65	82

Using Simpson's rule:

$$\text{Required Area} = \int_1^9 \left(x^2+1\right)dx$$

$$=\frac{1}{3}\left[\left(2+82\right)+2\left(10+26+50\right)+4\left(5+17+37+65\right)\right]$$

$$=\frac{1}{3}\left[84+172+496\right]$$

$$=\frac{752}{3}=250.67$$